Estimation non paramétrique de la fonction de Gerber–Shiu dans le modèle de Cramér–Lundberg

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Overview

1. The Cramér–Lundberg risk model
2. Estimation of the ruin probability
3. The Gerber–Shiu function
4. Laguerre deconvolution estimator
5. Laguerre–Fourier estimator

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The compound Poisson risk model [Asmussen and Albrecher, 2010]

Let $(U_t)_{t \geqslant 0}$ be the reserve process of an insurance company. In the compound Poisson risk model, this process is given by:

$$U_t = u + ct - \sum_{i=1}^{N_t} X_i,$$

where:

- $u \geqslant 0$ is the initial reserve,
- $c > 0$ is the premium rate,
- the claim number process $(N_t)_{t \geqslant 0}$ is a homogeneous Poisson process with intensity λ,
- the individual claim sizes $(X_i)_{i \geqslant 1}$ are positive, i.i.d. with density f and mean μ, independent of $(N_t)_{t \geqslant 0}$.

The ruin probability

We denote the time of ruin by $\tau \coloneqq \inf\{t \geqslant 0 \mid U_t < 0\} \in \mathbb{R}_+ \cup \{\infty\}$. We are interested in the ruin probability of the process $(U_t)_{t \geqslant 0}$ as a function of the initial reserve:

$$\phi(u) \coloneqq \mathbb{P}[\tau < \infty \mid U_0 = u].$$

Assumption (Safety Loading Condition)

A1 We assume that $c > \lambda\mu$. Introducing the parameter $\theta \coloneqq \frac{\lambda\mu}{c}$, the previous condition is equivalent to $\theta < 1$.

Under the SLC, we have $\phi(u) < 1$ for all $u \geqslant 0$.

The Pollaczeck–Khinchine formula

Theorem

Let $S(x) \coloneqq \mathbb{P}[X > x]$ *be the survival function of the* $(X_i)_{i\geqslant 1}$*. Under the SLC, the ruin probability is given by the formula:*

$$\phi(u) = (1-\theta)\sum_{k=1}^{+\infty}\theta^k H_k(u), \quad H_k(u) = \frac{1}{\mu^k}\int_u^{+\infty} S^{*k}(x)\,\mathrm{d}x.$$

Corollary

The ruin probability satisfies the renewal equation:

$$\phi = \phi * g + h,$$

where $g(x) \coloneqq \frac{\lambda}{c}S(x)$ *and* $h(u) \coloneqq \frac{\lambda}{c}\int_u^{+\infty} S(x)\,\mathrm{d}x$.

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Estimand and observations

We wish to estimate the ruin probability function ϕ.
We assume that the premium rate c is known. The parameters λ and μ may be assumed to be known or not.
Different observation setting can be considered:

1. We observe an i.i.d. sample $X_1, \ldots, X_n$ with distribution f.
2. We observe a trajectory of the process $(U_t)_{t\in[0,T]}$ on the finite interval $[0, T]$.
3. We observe discrete values $(U_{k\Delta})_{k=1,\ldots,n}$ of the reserve process, where $\Delta > 0$ is the sampling interval.

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Point estimation

- [Frees, 1986] constructs a Monte-Carlo estimator of $\phi(u)$ and shows its consistency.
- [Hipp, 1989] constructs a plug-in estimator from the Pollaczeck–Khinchine formula by replacing unknown quantities by empirical ones. He proves the asymptotic normality of his estimator.
- [Croux and Veraverbeke, 1990] use a similar estimator, but constructed as a linear combination of U-statistics, and show its asymptotic normality.

Functional estimation: plug-in empirical estimator [Pitts, 1994] [Politis, 2003]

These papers also consider a plug-in estimator using the Pollaczek–Khinchine formula, but they study its behavior as an element of a functional space.

Definition

Let D be the space of càdlàg functions on $[0, +\infty]$. For $\alpha \geqslant 0$, let D_α be the set of functions f such that $(1+u)^\alpha f$ can be extended as an element of D. We equip the space D_α with the norm:

$$\|f\|_\alpha := \sup_{u \in \mathbb{R}_+} \left|(1+u)^\alpha f(u)\right|.$$

Theorem

Let $\alpha \geqslant 0$. If $\mathbb{E}[X^{1+\alpha}]$ is finite, then he have:

$$\|\hat{\phi}_n - \phi\|_\alpha \xrightarrow[n\to\infty]{\text{a.s.}} 0.$$

Theorem

Let $\alpha' > \alpha \geqslant 0$. If $\mathbb{E}[X^{2(1+\alpha')}]$ is finite, then we have in the space D_α:

$$\sqrt{n}\,(\hat{\phi}_n - \phi) \xrightarrow[n\to\infty]{\text{d}} \mathcal{Z},$$

where $\mathcal{Z}$ is a zero mean Gaussian process.

They use this result to obtain confidence regions for ϕ.

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The Gerber–Shiu function

The Gerber–Shiu function, also called the *Expected Discounted Penalty Function (EDPF)*, is defined as:

$$\phi(u) \coloneqq \mathbb{E}\Big[\mathrm{e}^{-\delta\tau} w(U_{\tau^-}, |U_\tau|)\,\mathbf{1}_{\{\tau<\infty\}} \,\Big|\, U_0 = u\Big],$$

where $\delta \geqslant 0$ is a discounting force of interest, and $w\colon \mathbb{R}_+^2 \to \mathbb{R}_+$ is a penalty function.

Example

1. $\delta = 0$ and $w(x, y) = 1$, $\phi(u)$ is the ruin probability.
2. $\delta > 0$ and $w(x, y) = 1$, $\phi(u)$ is the Laplace transform of τ, evaluated at δ.
3. $\delta = 0$ and $w(x, y) = x + y$, $\phi(u)$ is the expected jump size causing the ruin.

Renewal equation

Theorem ([Gerber and Shiu, 1998])

Under Assumption A1 (SLC), the EDPF satisfies the equation:

$$\phi = \phi * g + h,$$

with:

$$g(x) := \frac{\lambda}{c} \int_x^{+\infty} \mathrm{e}^{-\rho_\delta (y-x)} f(y) \,\mathrm{d}y,$$
$$h(u) := \frac{\lambda}{c} \int_u^{+\infty} \mathrm{e}^{-\rho_\delta (x-u)} \left(\int_x^{+\infty} w(x, y-x) f(y) \,\mathrm{d}y \right) \mathrm{d}x,$$

and ρ_δ the non-negative solution of the Lundberg equation:

$$cs - \lambda\big(1 - \mathcal{L}f(s)\big) = \delta.$$

When $\delta = 0$, we have $\rho_\delta = 0$ as well.

Estimand and observations

We wish to estimate the Gerber–Shiu function ϕ.
We assume that the premium rate c is known. The parameters λ and μ are assumed to be unknown.
Different observation setting can be considered:

1. We observe an i.i.d. sample $X_1, \dots, X_n$ with distribution f.
2. We observe a trajectory of the process $(U_t)_{t \in [0,T]}$ on the finite interval $[0, T]$.
3. We observe discrete values $(U_{k\Delta})_{k=1,\dots,n}$ of the reserve process, where $\Delta > 0$ is the sampling interval.

Estimation strategy in a nutshell

1. We have:

$$g(x) = \frac{\lambda}{c}\,\mathbb{E}\Big[\mathrm{e}^{-\rho_\delta(X-x)}\mathbf{1}_{\{X>x\}}\Big],$$
$$h(u) = \frac{\lambda}{c}\,\mathbb{E}\bigg[\bigg(\int_u^X \mathrm{e}^{-\rho_\delta(x-u)} w(x, X-x)\,\mathrm{d}x\bigg)\mathbf{1}_{\{X>u\}}\bigg],$$

   with ρ_δ solution of $cs - \lambda(1 - \mathbb{E}[\mathrm{e}^{-sX}]) = \delta$. These quantities can be estimated from the observations.

2. Once we have estimated g and h, we solve the equation $\phi = \phi * g + h$ to estimate ϕ.

Following the work of [Comte et al., 2017] and [Mabon, 2017], [Zhang and Su, 2018] estimate g and h by projection on the Laguerre basis.

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Laguerre basis decomposition

The Laguerre functions $(\psi_k)_{k\in\mathbb{N}}$ are defined as:

$$\forall x \in \mathbb{R}_+, \quad \psi_k(x) \coloneqq \sqrt{2}\, L_k(2x)\, \mathrm{e}^{-x}, \quad L_k(x) \coloneqq \sum_{j=0}^{k} \binom{k}{j} \frac{(-x)^j}{j!}.$$

The Laguerre functions form a basis of $\mathrm{L}^2(\mathbb{R}_+)$. We decompose ϕ, g and h on this basis:

$$\phi = \sum_{k=0}^{+\infty} a_k\, \psi_k, \qquad g = \sum_{k=0}^{+\infty} b_k\, \psi_k, \qquad h = \sum_{k=0}^{+\infty} c_k\, \psi_k,$$

$$a_k = \langle \phi, \psi_k \rangle, \qquad b_k = \langle g, \psi_k \rangle, \qquad c_k = \langle h, \psi_k \rangle.$$

Estimation of g and h

Assumption

A2 $\int_0^{+\infty}(1+x)\int_x^{\infty} w(x,y-x)f(y)\,\mathrm{d}y\,\mathrm{d}x$ is finite. ($\implies h \in \mathrm{L}^2(\mathbb{R}_+)$)

A3 Let $W(X) \coloneqq \int_0^X \left(\int_u^X w(x,X-x)\,\mathrm{d}x\right)^2 \mathrm{d}u$. If $\delta = 0$ we assume that $\mathbb{E}[W(X)]$ is finite, if $\delta > 0$ we assume that $\mathbb{E}[W(X)^2]$ is finite.

The coefficients of g and h are given by:

$$b_k = \frac{\lambda}{c}\,\mathbb{E}\left[\int_0^X \mathrm{e}^{-\rho_\delta(X-x)}\psi_k(x)\,\mathrm{d}x\right],$$
$$c_k = \frac{\lambda}{c}\,\mathbb{E}\left[\int_0^X \left(\int_u^X \mathrm{e}^{-\rho_\delta(x-u)}w(x,X-x)\,\mathrm{d}x\right)\psi_k(u)\,\mathrm{d}u\right].$$

We estimate the coefficients of g and h by empirical means:

$$\hat{b}_k = \frac{1}{cT} \sum_{i=1}^{N_T} \int_0^{X_i} \mathrm{e}^{-\hat{\rho}_\delta (X_i - x)} \, \psi_k(x) \, \mathrm{d}x,$$

$$\hat{c}_k = \frac{1}{cT} \sum_{i=1}^{N_T} \int_0^{X_i} \left(\int_u^{X_i} \mathrm{e}^{-\hat{\rho}_\delta (x-u)} \, w(x, X_i - x) \, \mathrm{d}x \right) \psi_k(u) \, \mathrm{d}u,$$

with $\hat{\rho}_\delta$ the non-negative solution of the empirical Lundberg equation:

$$cs - \frac{N_T}{T} \left(1 - \frac{1}{N_T} \sum_{i=1}^{N_T} \mathrm{e}^{-sX_i} \right) = \delta.$$

For $m \in \mathbb{N}_+$, the projection estimators of g and h are:

$$\hat{g}_m := \sum_{k=0}^{m-1} \hat{b}_k \, \psi_k, \qquad \hat{h}_m := \sum_{k=0}^{m-1} \hat{c}_k \, \psi_k.$$

Bias-variance decomposition of the MISE

We quantify the quality of an estimator by its Mean Integrated Squared Error (MISE):

$$\mathbb{E}\|g - \hat{g}_m\|_{\mathsf{L}^2}^2.$$

The MISE of an estimator can be decomposed as the sum of a bias term and a variance term:

$$\begin{aligned}\mathbb{E}\|g - \hat{g}_m\|_{\mathsf{L}^2}^2 &= \big\|g - \Pi_{S_m}(g)\big\|_{\mathsf{L}^2}^2 + \mathbb{E}\big\|\hat{g}_m - \Pi_{S_m}(g)\big\|_{\mathsf{L}^2}^2 \\ &= \mathrm{dist}_{\mathsf{L}^2}^2(g, S_m) + \sum_{k=0}^{m-1} \mathbb{E}\Big[(\hat{b}_k - b_k)^2\Big],\end{aligned}$$

with $S_m \coloneqq \mathrm{Span}(\psi_0, \dots, \psi_{m-1})$.

Proposition

Under Assumptions A1, A2 and A3, if $\delta = 0$ then it holds:

$$\mathbb{E}\|g - \hat{g}_m\|_{\mathrm{L}^2}^2 \leqslant \mathrm{dist}_{\mathrm{L}^2}^2(g, S_m) + \frac{\lambda}{c^2 T}\mathbb{E}[X],$$
$$\mathbb{E}\|h - \hat{h}_m\|_{\mathrm{L}^2}^2 \leqslant \mathrm{dist}_{\mathrm{L}^2}^2(h, S_m) + \frac{\lambda}{c^2 T}\mathbb{E}[W(X)],$$

and if $\delta > 0$ then it holds:

$$\mathbb{E}\|g - \hat{g}_m\|_{\mathrm{L}^2}^2 \leqslant \mathrm{dist}_{\mathrm{L}^2}^2(g, S_m) + \frac{C(\lambda)}{c^2 T}\left(\mathbb{E}[X] + \frac{\mathbb{E}\left[X^2\right]^{\frac{1}{2}}}{(1-\theta)^2\delta^2}\right),$$
$$\mathbb{E}\|h - \hat{h}_m\|_{\mathrm{L}^2}^2 \leqslant \mathrm{dist}_{\mathrm{L}^2}^2(h, S_m) + \frac{C(\lambda)}{c^2 T}\left(\mathbb{E}[W(X)] + \frac{\mathbb{E}\left[W(X)^2\right]^{\frac{1}{2}}}{(1-\theta)^2\delta^2}\right),$$

where $C(\lambda) \asymp \lambda^2$.

Interlude: Laguerre deconvolution [Comte et al., 2017] [Mabon, 2017]

The Laguerre functions satisfy the relation:

$$\forall j, k \in \mathbb{N}, \quad \psi_j * \psi_k = 2^{-\frac{1}{2}}(\psi_{j+k} - \psi_{j+k+1}).$$

Using this relation, one can show that if f and g are two functions on $\mathbb{R}_+$ then their Laguerre coefficients satisfy:

$$c(f * g) = c(f) * \Delta(g), \quad \Delta_k(g) := \begin{cases} 2^{-\frac{1}{2}} \left(c_k(g) - c_{k-1}(g)\right) & : k \geqslant 1, \\ 2^{-\frac{1}{2}} c_0(g) & : k = 0. \end{cases}$$

If $\mathbf{c}_m(f)$ denotes the vector of the first m coefficients of f, we have:

$$\mathbf{c}_m(f * g) = \mathbf{G}_m \times \mathbf{c}_m(f), \quad \mathbf{G}_m := \begin{bmatrix} \Delta_0 & 0 & 0 & 0 & 0 \\ \Delta_1 & \Delta_0 & 0 & 0 & 0 \\ \Delta_2 & \Delta_1 & \Delta_0 & 0 & 0 \\ \cdots & \cdots & \cdots & \Delta_0 & 0 \\ \Delta_{m-1} & \Delta_{m-2} & \cdots & \cdots & \Delta_0 \end{bmatrix}.$$

Laguerre deconvolution estimator

If we use the convolution property of the Laguerre functions in the equation $\phi = \phi * g + h$, we obtain the following relation between the coefficients of ϕ, g and h:

$$\mathbf{c}_m = \mathbf{A}_m \times \mathbf{a}_m \iff \mathbf{a}_m = \mathbf{A}_m^{-1} \times \mathbf{c}_m,$$

with $\mathbf{A}_m \coloneqq \mathbf{Id}_m - \mathbf{G}_m$.

Assumption

A4 $(b_{k+1} - b_k)_{k \in \mathbb{N}} \in \ell^1(\mathbb{N})$.

Lemma

Under Assumption A1 and A4, we have $\|\mathbf{A}_m^{-1}\|_{\mathrm{op}} \leqslant \frac{2}{1 - \|g\|_{\mathbb{L}^1}} \leqslant \frac{2}{1-\theta}$.

For $\theta_0 < 1$ a truncation parameter, we estimate ϕ by:

$$\hat{\phi}_m := \sum_{k=0}^{m-1} \hat{a}_k \, \psi_k, \quad \hat{\mathbf{a}}_m := \widetilde{\mathbf{A}}_m^{-1} \times \hat{\mathbf{c}}_m, \quad \widetilde{\mathbf{A}}_m^{-1} := \hat{\mathbf{A}}_m^{-1} \mathbf{1}_{\left\{ \|\hat{\mathbf{A}}_m^{-1}\|_{\mathrm{op}} \leqslant \frac{2}{1-\theta_0} \right\}}.$$

Proposition

Under Assumptions A1, A2, A3, and A4, if $\theta < \theta_0$ then it holds:

$$\mathbb{E}\|\phi - \hat{\phi}_m\|_{\mathsf{L}^2}^2 \leqslant \mathrm{dist}_{\mathsf{L}^2}^2(\phi, S_m) + C\,\frac{m}{T},$$

where C is a constant depending on λ, c, θ, $\mathbb{E}[X]$, $\mathbb{E}[W(X)]$ and $\theta_0 - \theta$; and also δ, $\mathbb{E}[X^2]$, $\mathbb{E}[W(X)^2]$ if $\delta > 0$.

The Laguerre–Sobolev spaces [Bongioanni and Torrea, 2009]

Definition

For $s \in (0, +\infty)$, we define the Sobolev–Laguerre space as:

$$\mathrm{W}^s(\mathbb{R}_+) := \left\{ f \in \mathrm{L}^2(\mathbb{R}_+) \,\middle|\, \sum_{k \in \mathbb{N}} \langle f, \psi_k \rangle^2 \, k^s < +\infty \right\}.$$

Theorem ([Comte and Genon-Catalot, 2015])

Let $s \in \mathbb{N}_+$. A function f belongs to $\mathrm{W}^s(\mathbb{R}_+)$ iff:

1. *f admits derivatives up to order $s-1$, and $f^{(s-1)}$ is absolutely continuous;*
2. *$\forall k \in \{0, \dots, s-1\}$, $x^{\frac{k-1}{2}} \sum_{j=0}^{k+1} \binom{k+1}{j} f^{(j)} \in \mathrm{L}^2(\mathbb{R}_+)$.*

Rate of convergence

Theorem

We assume A1, A2, A3, A4, and we assume that $\theta < \theta_0$. If $\phi \in \mathrm{W}^s(\mathbb{R}_+)$, then choosing $m_{\text{opt}} \propto T^{\frac{1}{1+s}}$ yields:

$$\mathbb{E}\|\phi - \hat{\phi}_{m_{\text{opt}}}\|_{\mathrm{L}^2}^2 \lesssim T^{-\frac{s}{1+s}}.$$

- This method does not recover the rate T^{-1} for the ruin probability.
- The functions g and h are estimated with the rate T^{-1}, but the deconvolution step loses a factor m in the variance term.

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Laguerre–Fourier estimator [Dussap, 2022]

Since $\phi = \phi * g + h$, we have $\mathcal{F}\phi = \frac{\mathcal{F}h}{1-\mathcal{F}g}$. We compute the coefficients of ϕ using Plancherel theorem:

$$a_k = \langle \phi, \psi_k \rangle = \frac{1}{2\pi} \langle \mathcal{F}\phi, \mathcal{F}\psi_k \rangle = \frac{1}{2\pi} \left\langle \frac{\mathcal{F}h}{1-\mathcal{F}g}, \mathcal{F}\psi_k \right\rangle.$$

Definition

For $\hat{g}$ and $\hat{h}$ two estimators of g and h, and for θ_0 a truncation parameter, we estimate ϕ by:

$$\hat{\phi}_{m_1,\hat{g},\hat{h}} := \sum_{k=0}^{m_1-1} \hat{a}_{k,\hat{g},\hat{h}} \psi_k, \quad \hat{a}_{k,\hat{g},\hat{h}} := \frac{1}{2\pi} \left\langle \frac{\mathcal{F}\hat{h}}{1-\widetilde{\mathcal{F}g}}, \mathcal{F}\psi_k \right\rangle,$$

$$\widetilde{\mathcal{F}g} := (\mathcal{F}\hat{g})\mathbf{1}_{\{|\mathcal{F}\hat{g}|<\theta_0\}}.$$

Proposition

Under Assumption A1 and A2, if $\theta < \theta_0$ then it holds:

$$\|\phi - \hat{\phi}_{m_1,\hat{g},\hat{h}}\|^2_{\mathrm{L}^2} \leqslant \mathrm{dist}^2_{\mathrm{L}^2}(\phi, S_{m_1}) + \frac{2}{(1-\theta_0)^2}\|h - \hat{h}\|^2_{\mathrm{L}^2} + \frac{2\,\|h\|^2_{\mathrm{L}^1}}{(1-\theta_0)^2(1-\theta)^2}\left(1 + \frac{\|g\|^2_{\mathrm{L}^1}}{(\theta_0-\theta)^2}\right)\|g - \hat{g}\|^2_{\mathrm{L}^2}.$$

If we use the Laguerre projection estimators $\hat{g}_{m_2}$ and $\hat{h}_{m_3}$, we obtain the following result.

Corollary

Under Assumptions A1, A2 and A3, if $\theta < \theta_0$ then it holds:

$$\mathbb{E}\|\phi - \hat{\phi}_{m_1,m_2,m_3}\|^2_{\mathrm{L}^2} \leqslant \mathrm{dist}^2_{\mathrm{L}^2}(\phi, S_{m_1}) + C\left(\mathrm{dist}^2_{\mathrm{L}^2}(g, S_{m_2}) + \mathrm{dist}^2_{\mathrm{L}^2}(h, S_{m_3}) + \frac{1}{T}\right).$$

Rates of convergence

Theorem

We assume A1, A2, A3, and we assume that $\theta < \theta_0$. *If* $\phi \in \mathrm{W}^{s_1}(\mathbb{R}_+)$, $g \in \mathrm{W}^{s_2}(\mathbb{R}_+)$ *and* $h \in \mathrm{W}^{s_3}(\mathbb{R}_+)$, *then choosing* $m_i \geqslant T^{\frac{1}{s_i}}$ *for all* $i \in \{1, 2, 3\}$ *yields:*

$$\mathbb{E}\|\phi - \hat{\phi}_{m_1,m_2,m_3}\|_{\mathrm{L}^2}^2 \lesssim T^{-1}.$$

Conclusion and perspectives

- If ϕ belongs to a Sobolev–Laguerre space of regularity greater than 1, it is possible to estimate the EDPF with rate T^{-1}.
- The Laguerre deconvolution method fails to recover the parametric rate.
- The absence of a bias-variance compromise raises questions about how to perform a model selection procedure.
- The Laguerre–Fourier method could be extended to more general risk models.

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